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REVISITING INTEGRATION IN THE MATERIAL POINT METHOD

A SCHEME FOR EASIER SEPARATION AND LESS DISSIPATION

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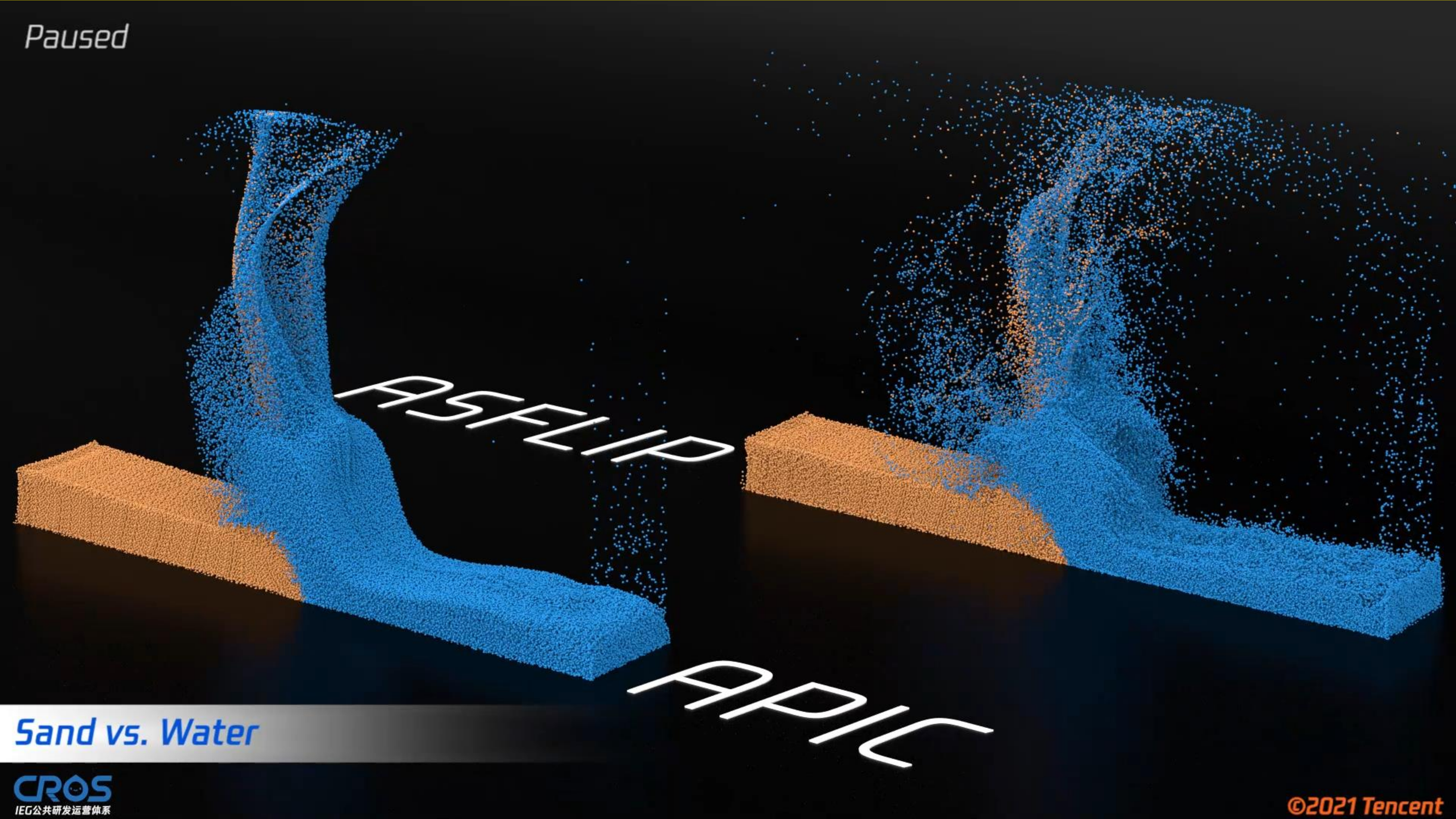
Paused
ASFLIP
APIC
Sand vs. Water
CROS
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MOTIVATION

WHY WE NEED A NEW INTEGRATION SCHEME?

A CASE IN PRODUCTION

- 93,014 strands
- 7,179,350 DoFs
- Required simulation cost
  - ~ 6s per frame (based on 60 frame per simulated second)
  - ~ 1h in total (10s simulation)

Simulate with GPU-MPM?

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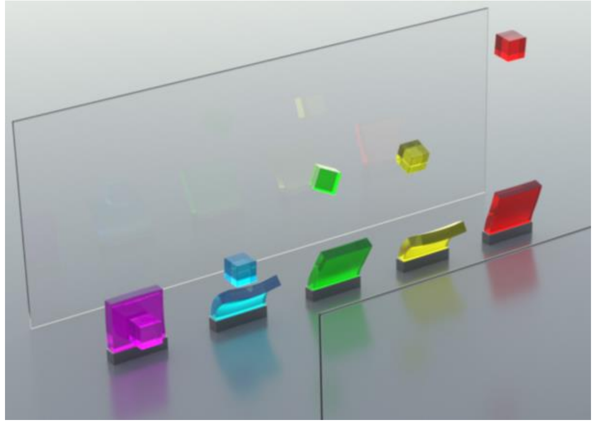
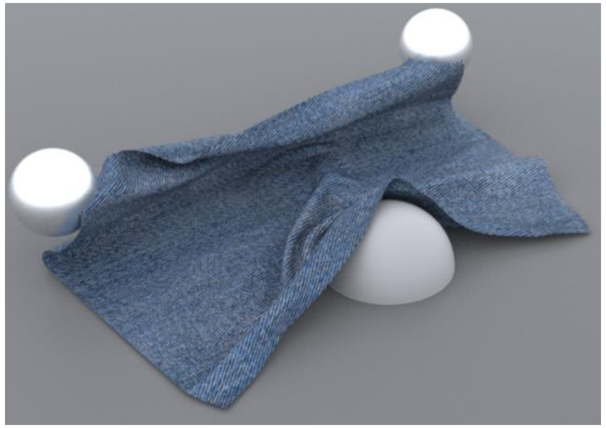
STUDIOS

PRIOR WORK OF MPM WITH LAGRANGIAN FORCES



[Jiang et al. 2015]

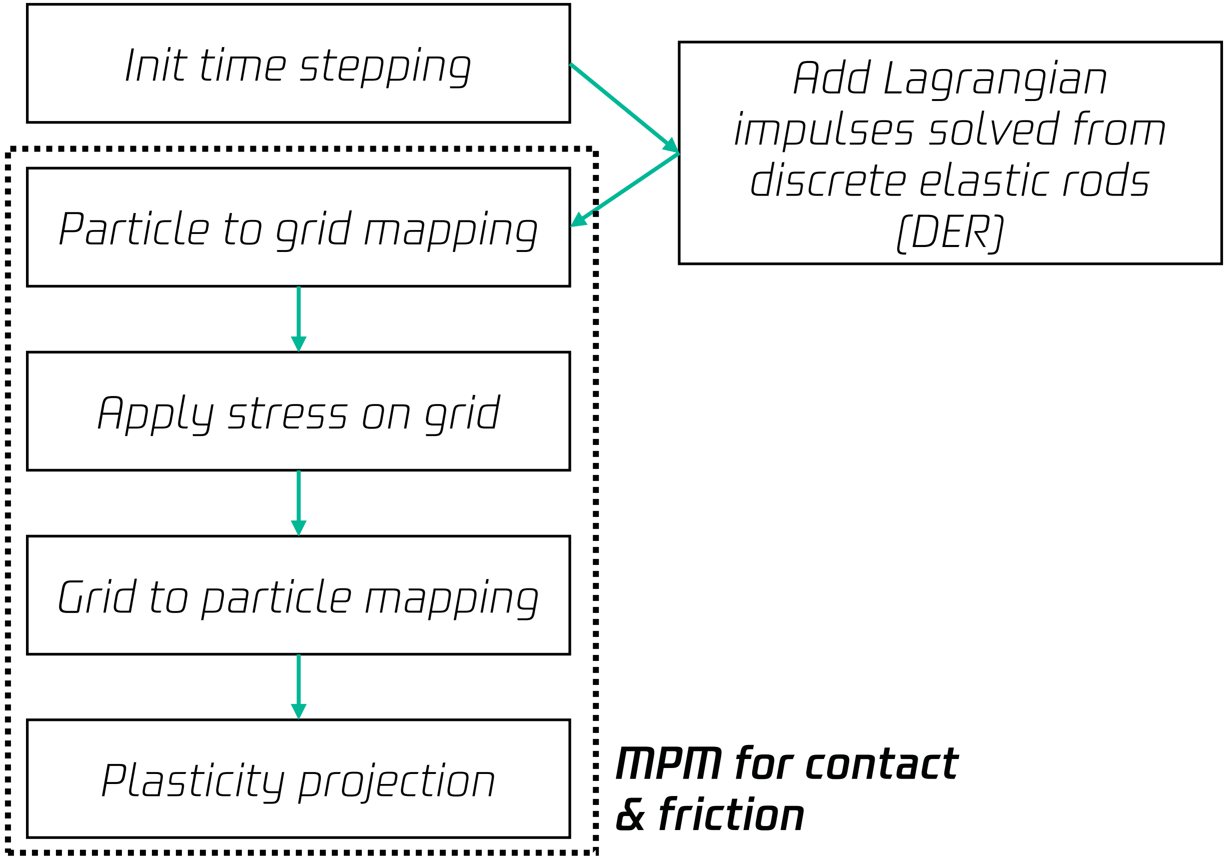
[Jiang et al. 2017]



[Guo et al. 2018]

[Han et al. 2019]

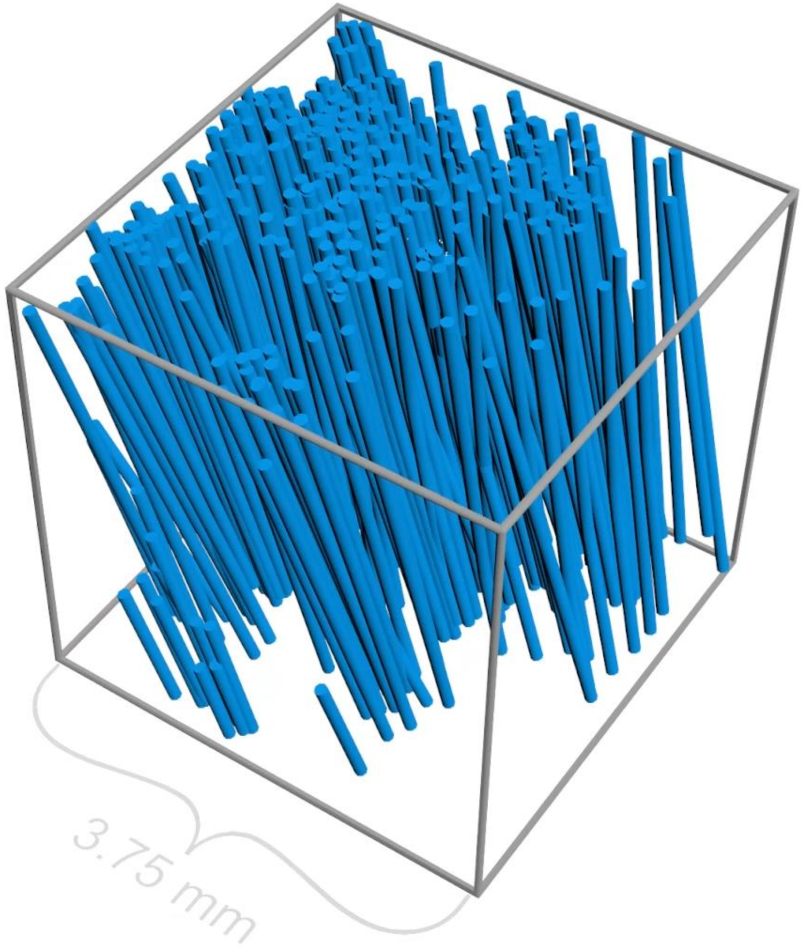
AN MPM HAIR SIMULATOR



Our MPM Hair Simulator

HAIRS SIMULATED WITH MLS-MPM + FLIP

HAIRS SIMULATED WITH MLS-MPM + FLIP

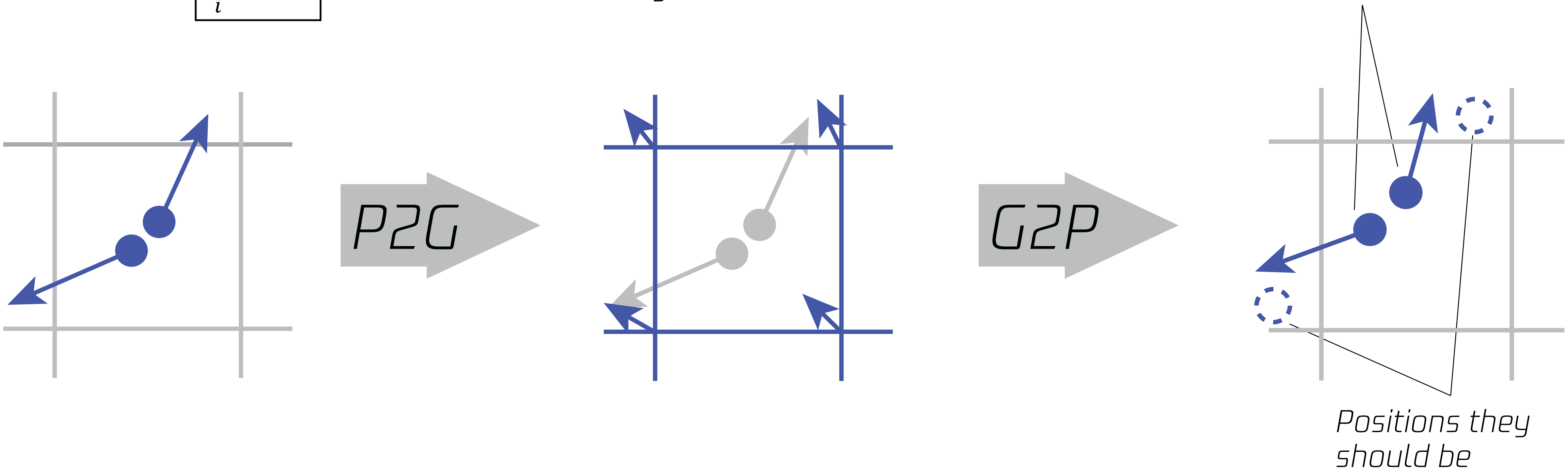


FLIP INTEGRATOR
[BRACKBILL & RUPPEL 1986]

Position update *during grid-to-particle (G2P) transfer:*

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \boxed{\sum_i w_{ip} \boldsymbol{v}_i^*}$$

Interpolated from current nodal velocity



PIC/APIC/POLYPIC INTEGRATORS

PIC grid-to-particle:
[Harlow et al. 1955]

APIC grid-to-particle:
(moving-least-square version)
[Jiang et al. 2015, Hu et al. 2018]

PolyPIC grid-to-particle:
[Fu et al. 2017]

Velocity Update

PIC:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^*$$

APIC:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^*$$

$$\mathbf{C}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* \left(x_i^n - x_p^n\right)^T \left(\mathbf{D}_p^n\right)^{-1}$$

PolyPIC:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^*$$

$$\mathbf{C}_p^{n+1} = \underset{\boldsymbol{c}}{\operatorname{argmin}} \sum_i m_{ip}^n \left| \boldsymbol{v}_i^* - \sum_r \sum_\alpha s_r\left(x_i - x_p^n\right) e_\alpha c_{pr\alpha}^{n+1} \right|^2$$

Position Update

PIC:

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

APIC:

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

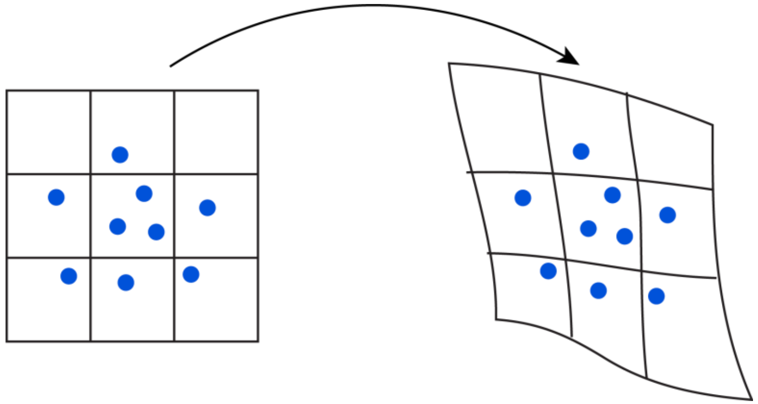
PolyPIC:

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

THE CONTINUUM ASSUMPTION

- *FLIP/xPIC assumes material* ***continuous in the entire domain****.*
- *Problem:*
  - *Particle position is* ***always*** *interpolated from deformed nodal positions*
  - *If any movement is* ***not capturable*** *by grid, it would be* ***neither capturable*** *by particles.*

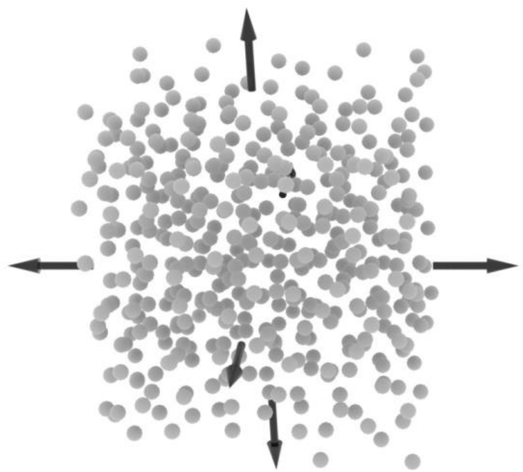
Continuous deformation Ψ



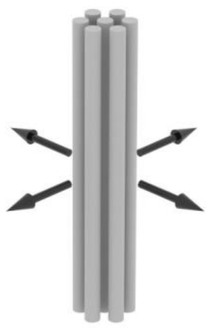
$$\frac{\partial \boldsymbol{x}}{\partial t} = \frac{\partial \Psi(X,t)}{\partial t} \approx \sum_i w_{ip} \boldsymbol{v}_i^*$$

$$\boldsymbol{x}_p^{n+1} = \sum_i w_{ip} \boldsymbol{x}_i^*$$

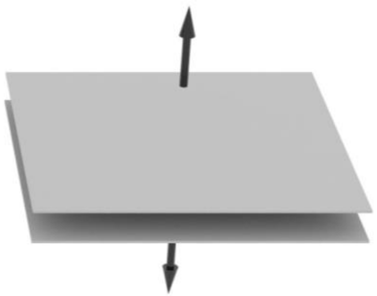
DISCREPANCY
BETWEEN RENDERING, EXPECTATION AND SIMULATION



Sand



Hairs



Clothes

Expected behavior: ***separate immediately*** *when pulled apart*

Rendering: ***discrete & separated*** *elements*

Simulation: ***continuum*** *that trap particles until they have no shared weights on a node*

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NFLIP

NATURALLY-MODIFIED **FLIP**

NFLIP: USE PARTICLE VELOCITY FOR ADVECTION

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FLIP *grid-to-particle:*

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* + \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n \right)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

Assume material continuous everywhere.

NFLIP *grid-to-particle:*

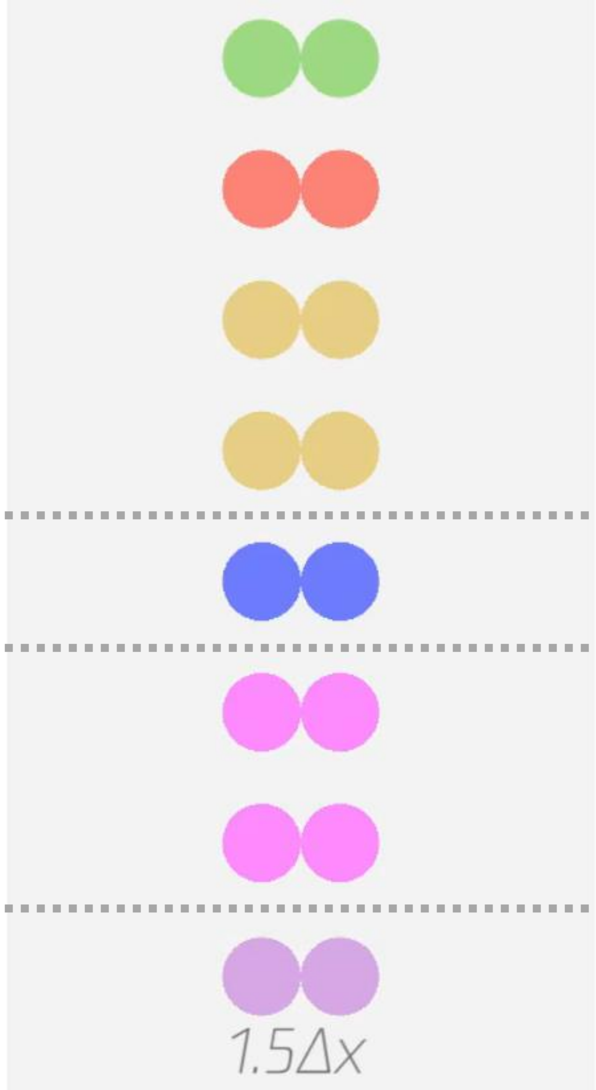
$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* + \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n \right)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \boldsymbol{v}_p^{n+1}$$

Completely ignore continuity.

NFLIP: USE PARTICLE VELOCITY FOR ADVECTION

Method		Observation
PIC		
APIC		
FLIP 0.99		
FLIP 0.995		dissipated, difficult to separate
Full FLIP		slow to separate, then accelerated
NFLIP 0.99		
NFLIP 0.995		dissipated, easy to separate
Ground Truth		



IGNORING CONTINUITY LEADS TO ERROR

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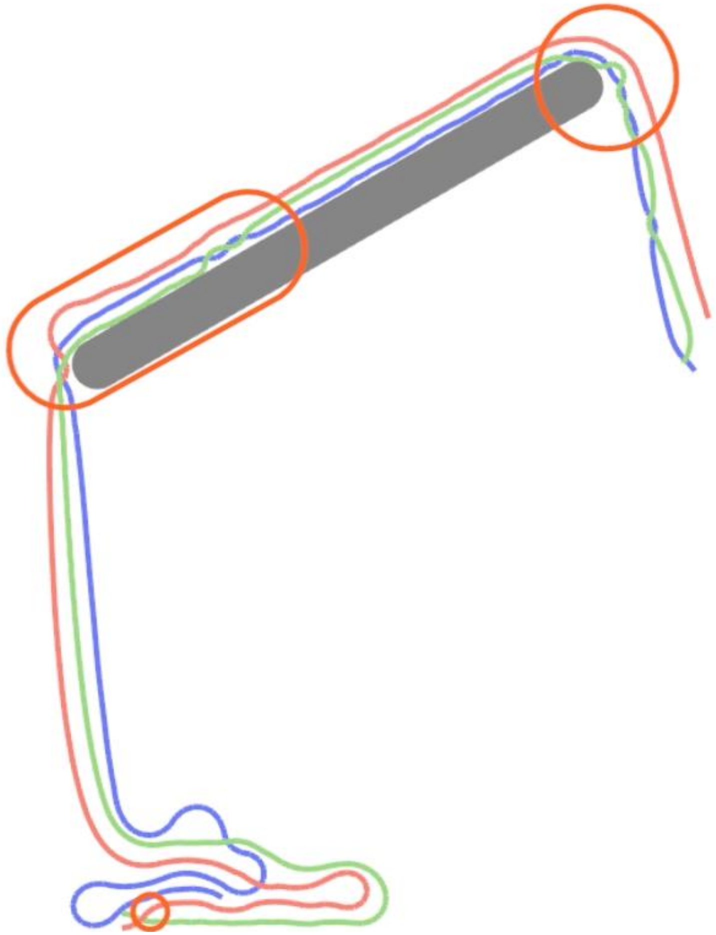
Ignoring continuity

Wrong deformation gradient

Wrong stress

Wrong contacts and frictions



SFLIP

A SEPARABLE FLIP

A SEPARABLE FLIP

FLIP grid-to-particle:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip}\boldsymbol{v}_i^* + \alpha\left(\boldsymbol{v}_p^n - \sum_i w_{ip}\boldsymbol{v}_i^n\right)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip}\boldsymbol{v}_i^*$$

SFLIP grid-to-particle:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip}\boldsymbol{v}_i^* + \alpha\left(\boldsymbol{v}_p^n - \sum_i w_{ip}\boldsymbol{v}_i^n\right)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \left[\sum_i w_{ip}\boldsymbol{v}_i^* + \beta_p\alpha\left(\boldsymbol{v}_p^n - \sum_i w_{ip}\boldsymbol{v}_i^n\right)\right]$$

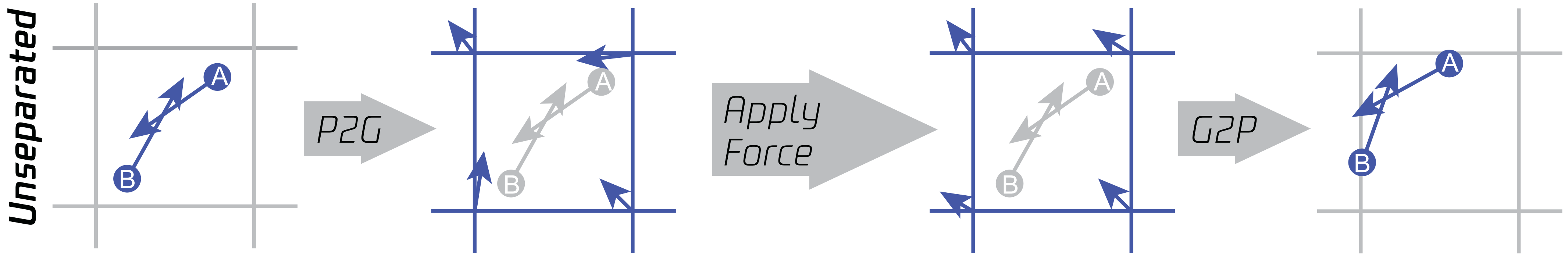
β_p: the trap-breaking ratio

FLIP: $\forall p, \beta_p = 0$

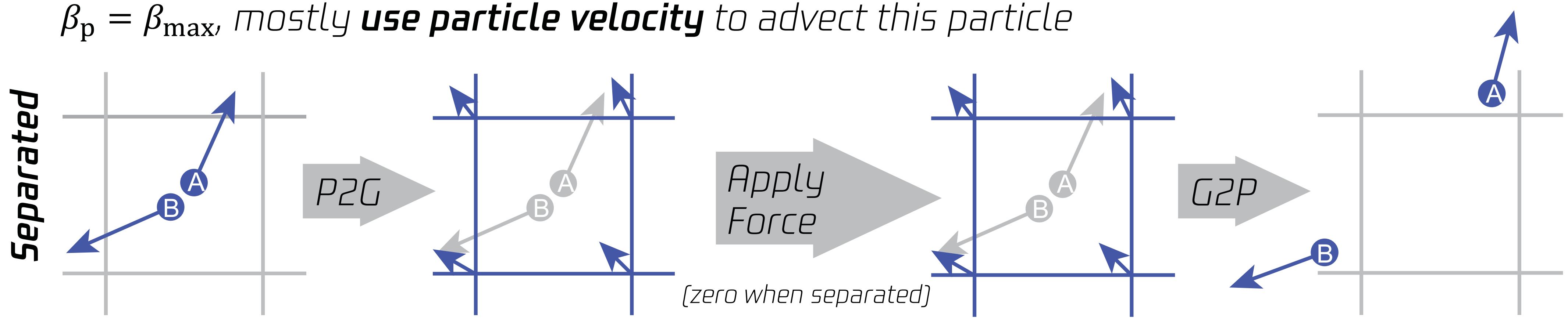
NFLIP: $\forall p, \beta_p = 1$

THE TRAP-BREAKING RATIO β_p

$\beta_p = \beta_{min}$, *mostly use interpolated nodal velocity to advect this particle*



$\beta_p = \beta_{max}$, *mostly* ***use particle velocity*** *to advect this particle*



DETERMINE SEPARATION THROUGH VOLUME RATIO

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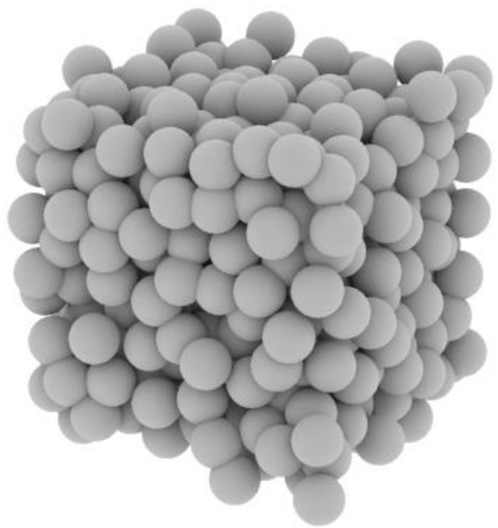

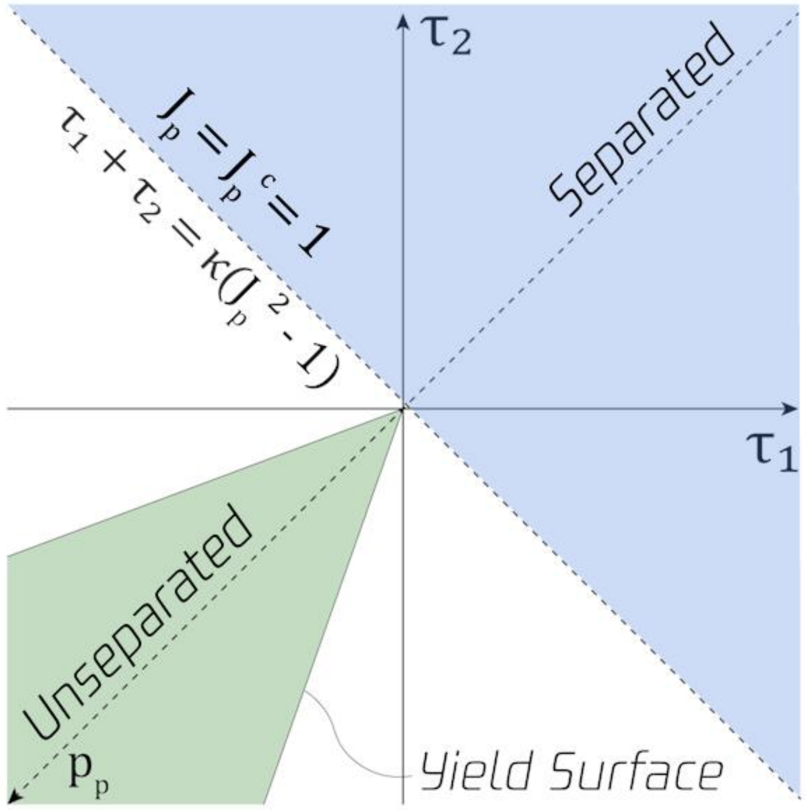

τ_1, τ_2: stress

J_p: volume ratio

p_p: pressure

κ: bulk modulus

J_p^c: critical volume ratio

DETERMINE SEPARATION THROUGH VOLUME RATIO

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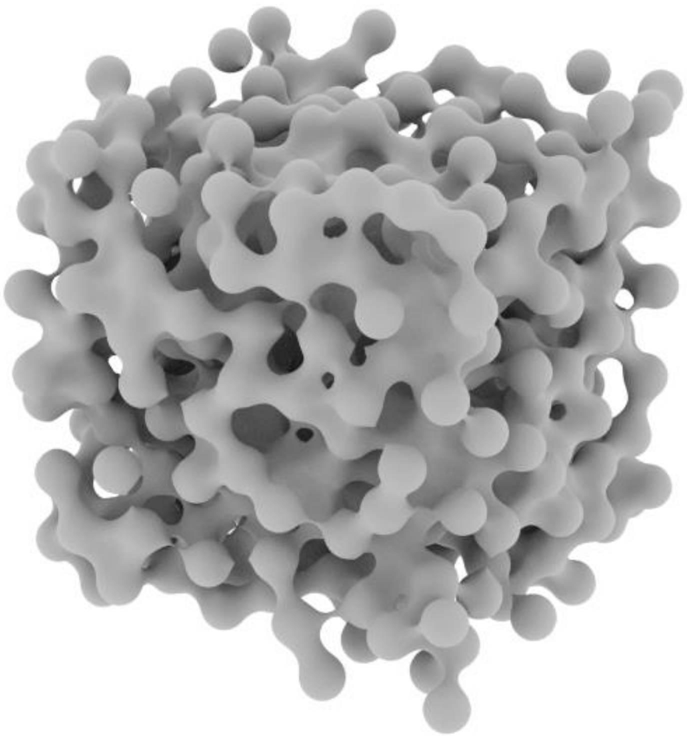

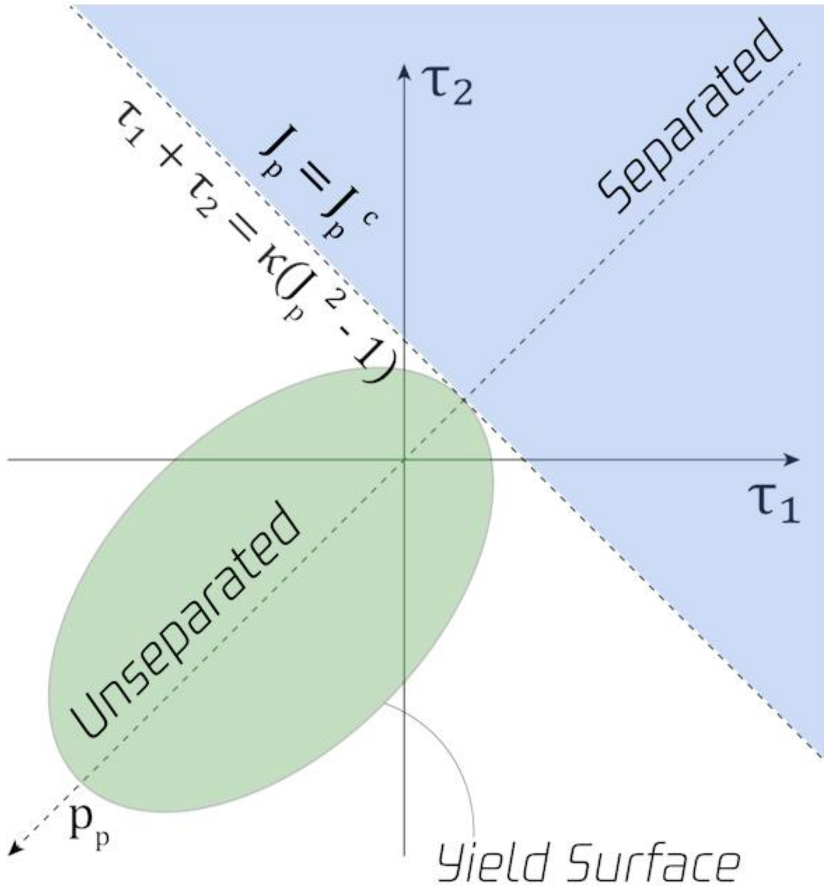

τ_1, τ_2: stress

J_p: volume ratio

p_p: pressure

κ: bulk modulus

J_p^c: critical volume ratio

A SEPARABLE FLIP - SUMMARIZED

SFLIP grid-to-particle:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip}\boldsymbol{v}_i^* + \alpha\left(\boldsymbol{v}_p^n - \sum_i w_{ip}\boldsymbol{v}_i^n\right)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t\left[\sum_i w_{ip}\boldsymbol{v}_i^* + \beta_p\alpha\left(\boldsymbol{v}_p^n - \sum_i w_{ip}\boldsymbol{v}_i^n\right)\right]$$

$$\beta_p = \begin{cases} 0, & \text{in boundary} \\ \beta_{\min}, & J_p < J_p^c \\ \beta_{\max}, & J_p \geq J_p^c \end{cases}$$

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SFLIP

DIDACTIC EXAMPLES

TWO PERFECTLY SMOOTH FINITE ELEMENT SQUARES

NO FORCE APPLIED FROM ONE SQUARE TO ANOTHER

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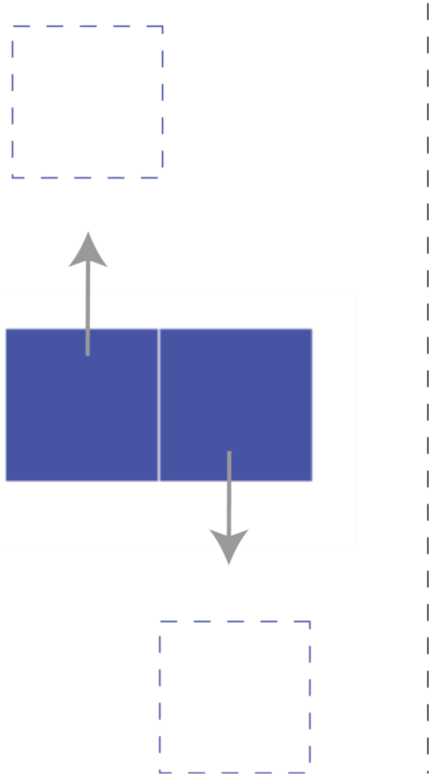
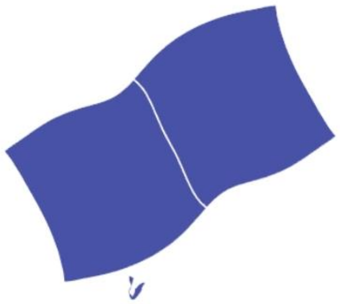
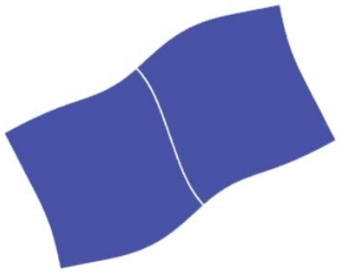
Ground Truth



FLIP

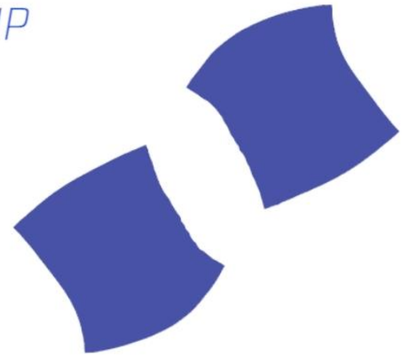


APIC

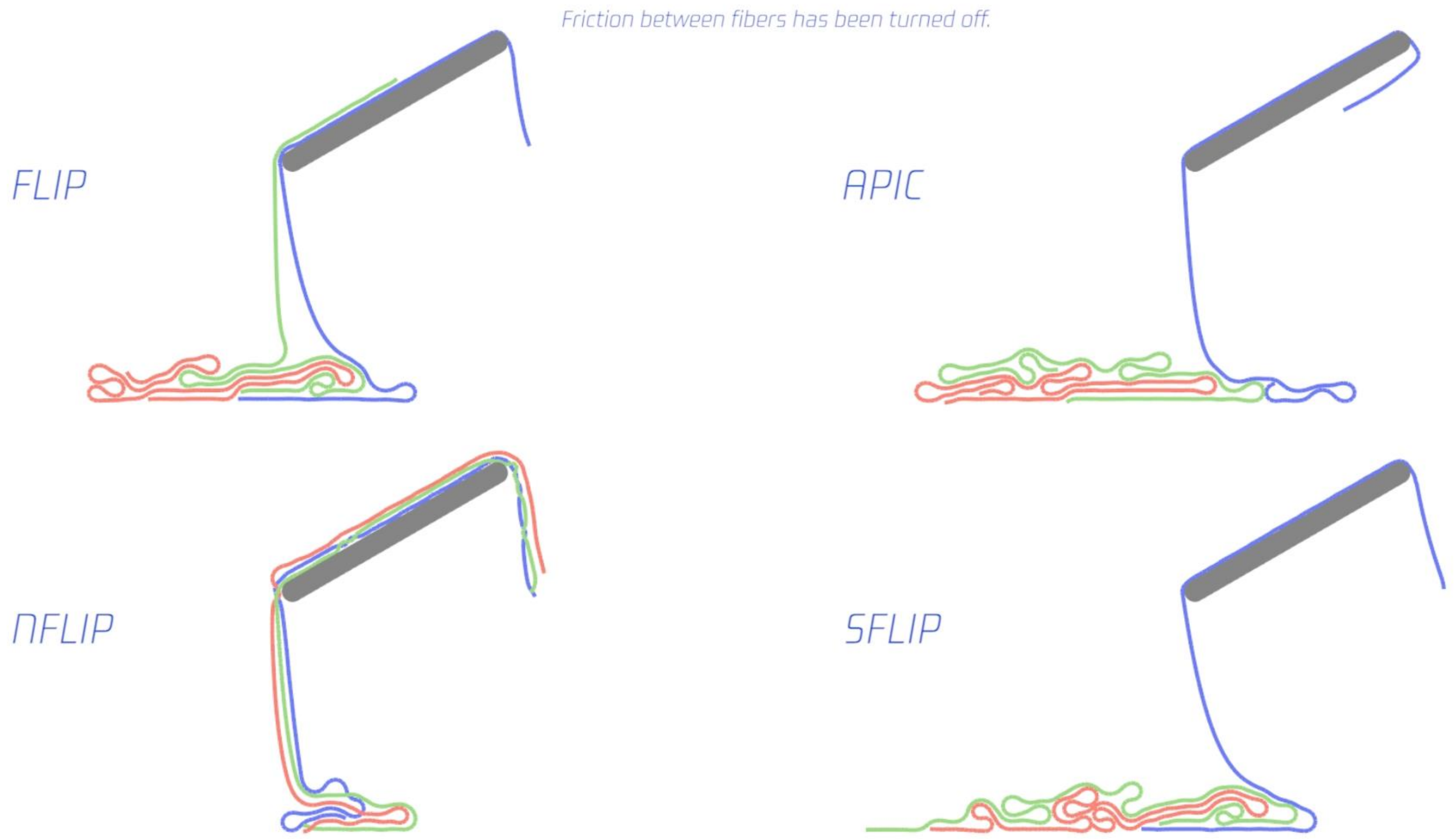


NFLIP

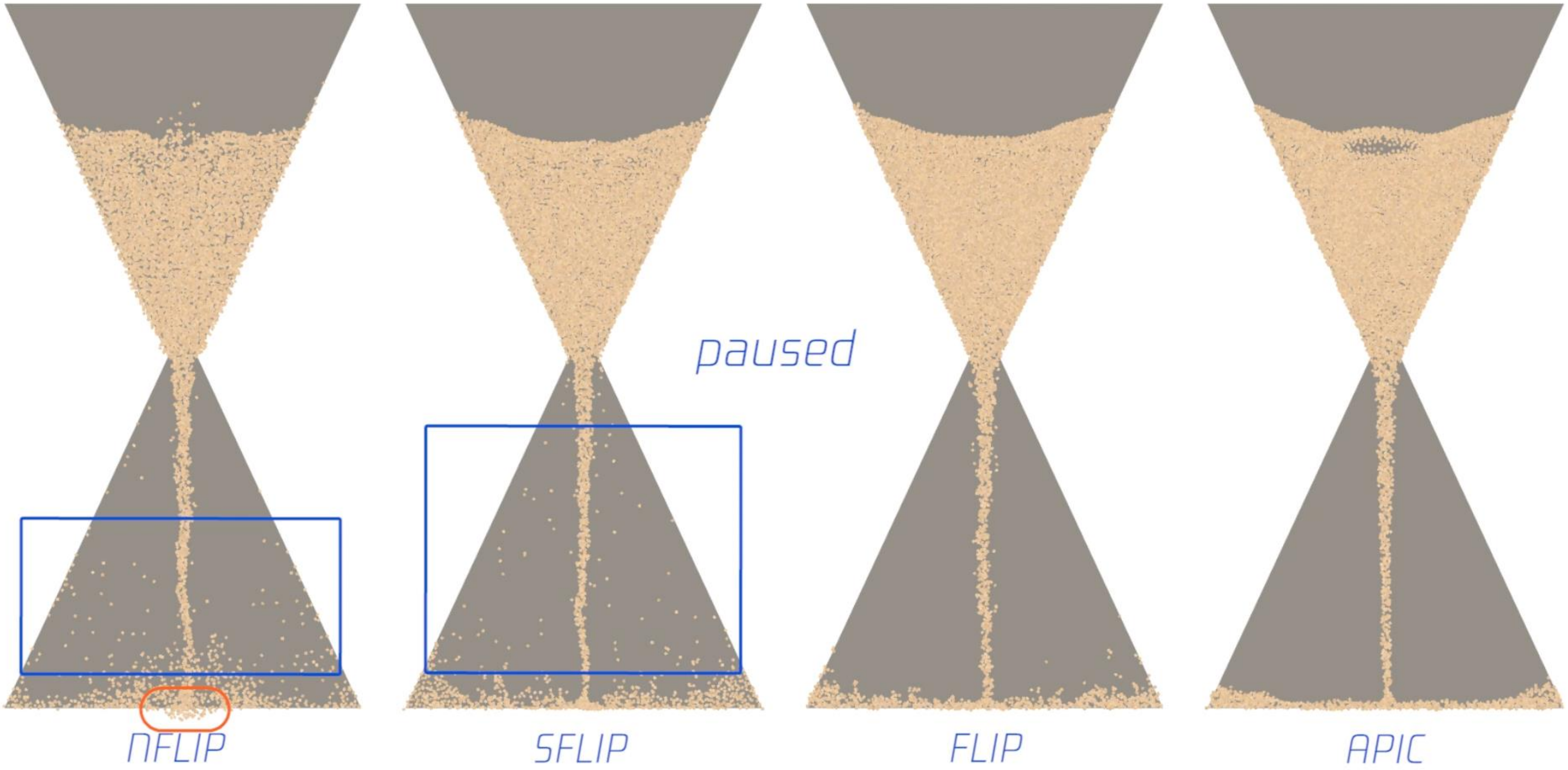
SFLIP



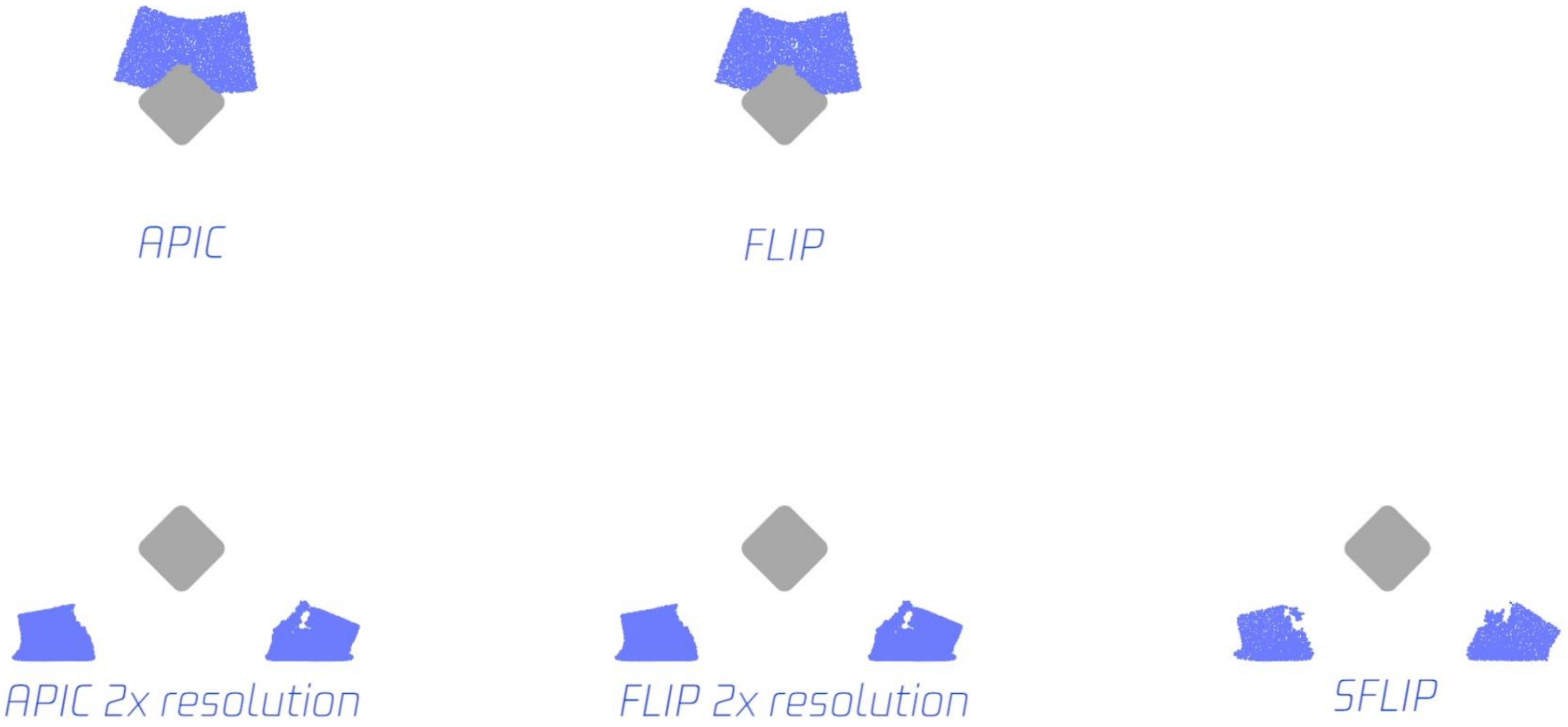
2D FIBERS


Friction between fibers has been turned off.
FLIP
APIC
NFLIP
SFLIP

2D HOURGLASS



2D CLAY FRACTURE



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ASFLIP

AFFINE-AUGMENTED SEPARABLE FLIP

NAÏVE MODIFICATION TO IMPROVE APIC

Particle-to-grid transfer:

$$m_i^n \boldsymbol{v}_i^n = \sum_p w_{ip} m_p \left[\boldsymbol{v}_p^n + \mathbf{C}_p^n \left(\boldsymbol{x}_i^n - \boldsymbol{x}_p^n\right)\right]$$

APIC grid-to-particle transfer:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^*$$

$$\mathbf{C}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* \left(x_i^n - x_p^n\right)^T \left(\mathbf{D}_p^n\right)^{-1}$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

Add positional adjustment

ASPIC grid-to-particle transfer:

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^*$$

$$\mathbf{C}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* \left(x_i^n - x_p^n\right)^T \left(\mathbf{D}_p^n\right)^{-1}$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \left[\sum_i w_{ip} \boldsymbol{v}_i^* + \beta_p \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n\right)\right]$$

$$\beta_p = \begin{cases} 0, & \text{in boundary or source} \\ \beta_{\min}, & J_p < J_p^c \\ \beta_{\max}, & J_p \geq J_p^c \end{cases}$$

ASPIC

APIC + POSITIONAL ADJUSTMENT

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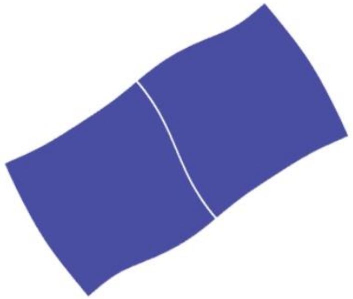

ASPIC doesn't work.

Assume zero force applied (i.e., $\boldsymbol{v}_i^* = \boldsymbol{v}_i^n$):

$$\boxed{\boldsymbol{v}_p^{n+1} = \sum_i w_{ip}^n \boldsymbol{v}_i^*}$$

Root cause: PIC-style velocity update never preserves particle velocity from previous steps

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \left[\sum_i w_{ip}^n \boldsymbol{v}_i^* + \beta_p \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip}^n \boldsymbol{v}_i^n \right) \right]$$

$$\Downarrow$$

$$\underbrace{\boldsymbol{x}_p^{n+1}}_{\text{Only depends on nodal velocity}} = \boldsymbol{x}_p^n + \Delta t \sum_i \left[(1 - \beta_p \alpha) w_{ip}^n \underbrace{\boldsymbol{v}_i^n}_{\text{Nodal velocity at } t^n} + \beta_p \alpha w_{ip}^{n-1} \underbrace{\boldsymbol{v}_i^{n-1}}_{\text{Nodal velocity at } t^{n-1}} \right]$$

AFFINE-AUGMENTED FLIP (AFLIP)

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AFLIP particle-to-grid transfer (P2G):

$$m_i^n \boldsymbol{v}_i^n = \sum_p w_{ip} m_p \left[\boldsymbol{v}_p^n + \mathbf{C}_p^n \left(\boldsymbol{x}_i^n - \boldsymbol{x}_p^n \right) \right]$$

→ Preserves affine momentum during P2G

AFLIP grid-to-particle transfer (G2P):

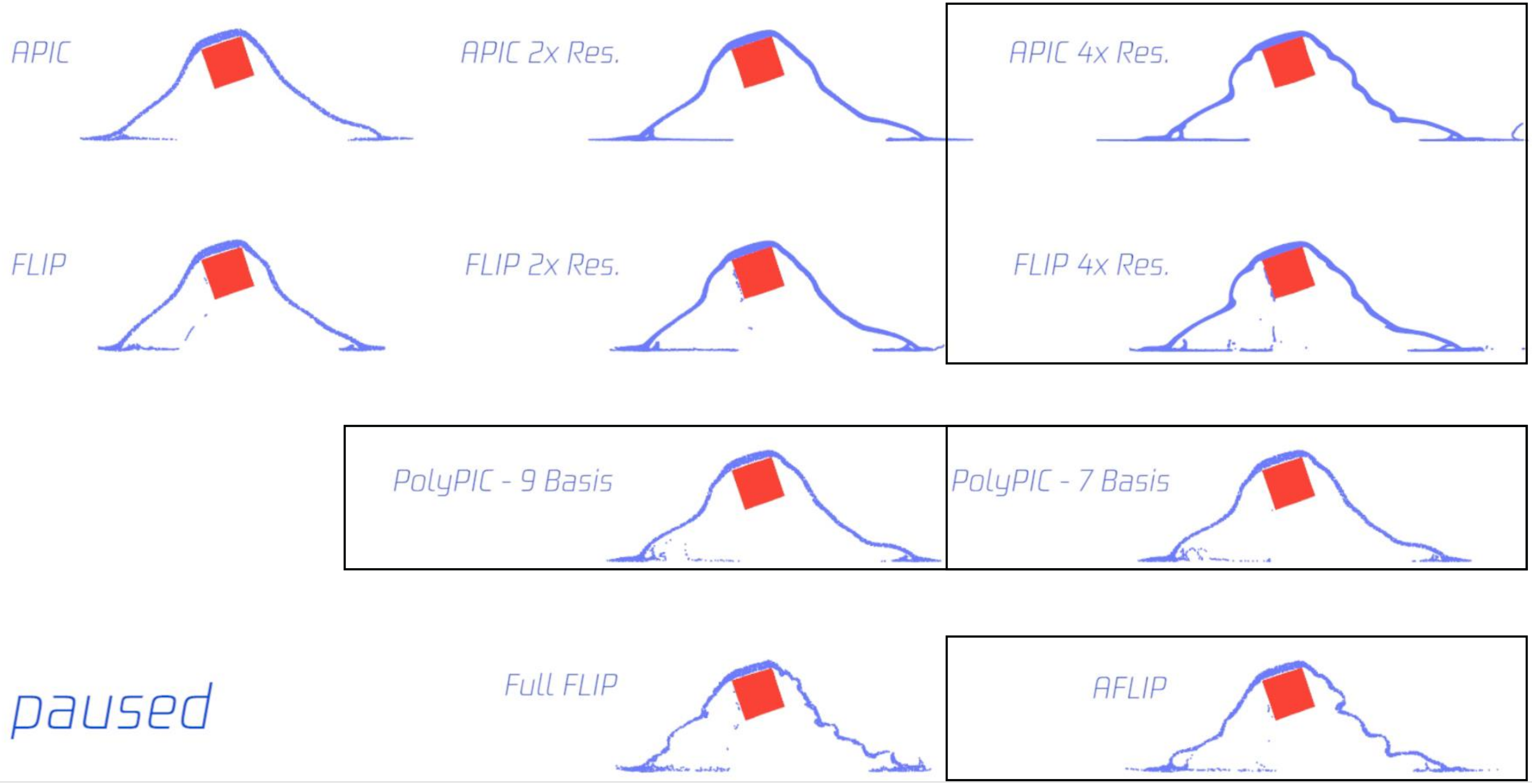
$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* + \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n \right)$$

→ Preserves high-frequency momentum during G2P (damped through $\alpha < 1$)

$$\mathbf{C}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* \left(x_i^n - x_p^n \right)^T \left(\mathbf{D}_p^n \right)^{-1}$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \sum_i w_{ip} \boldsymbol{v}_i^*$$

2D WEAKLY-COMPRESSIBLE LIQUID



ASFLIP particle-to-grid transfer (P2G):

$$m_i^n \boldsymbol{v}_i^n = \sum_p w_{ip} m_p \left[\boldsymbol{v}_p^n + \mathbf{C}_p^n \left(\boldsymbol{x}_i^n - \boldsymbol{x}_p^n\right)\right]$$

Preserves affine momentum during P2G

ASFLIP grid-to-particle transfer (G2P):

$$\boldsymbol{v}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* + \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n\right)$$

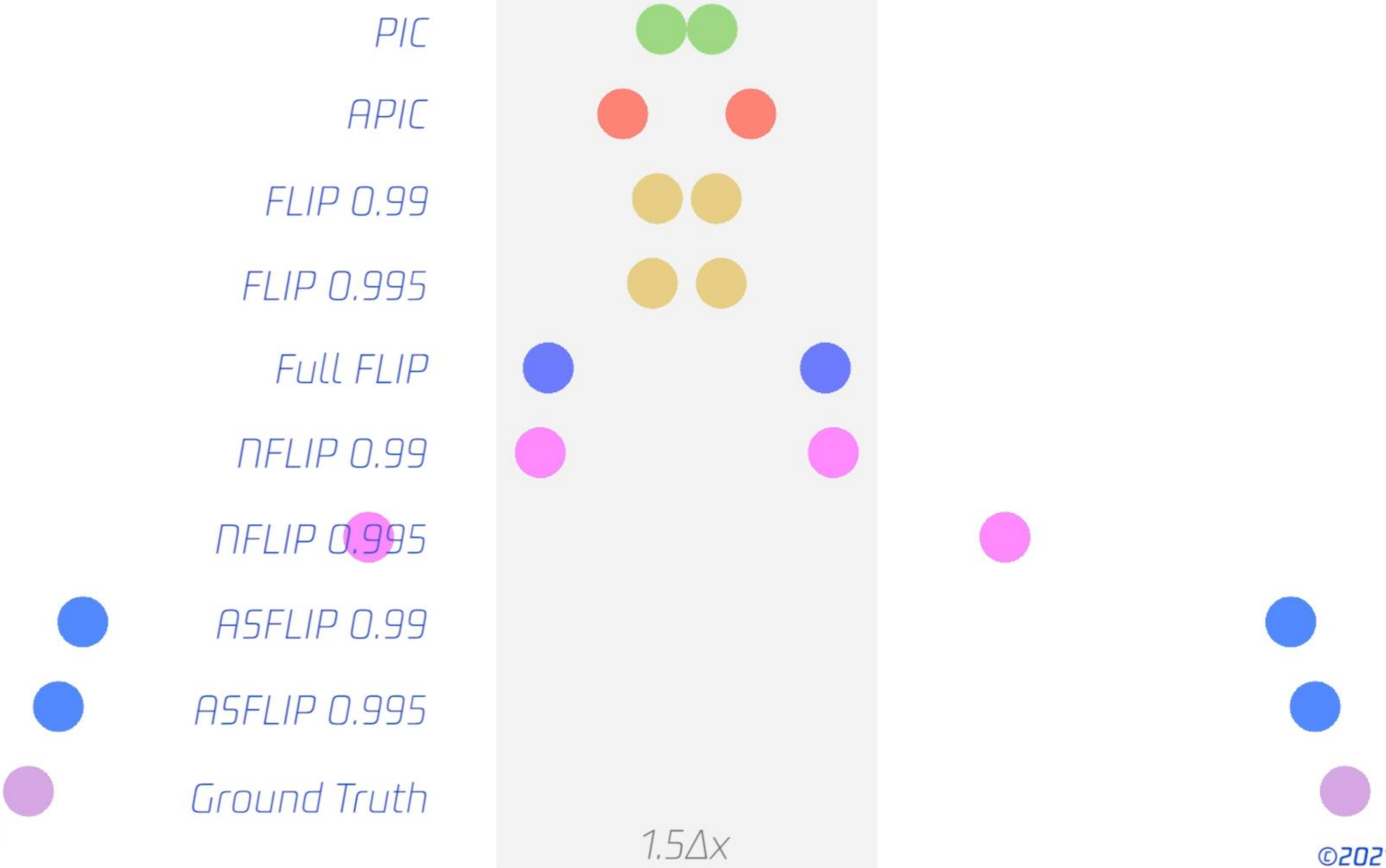
Preserves high-frequency momentum during G2P
(damped through $\alpha < 1$)

$$\mathbf{C}_p^{n+1} = \sum_i w_{ip} \boldsymbol{v}_i^* \left(x_i^n - x_p^n\right)^T \left(\mathbf{D}_p^n\right)^{-1}$$

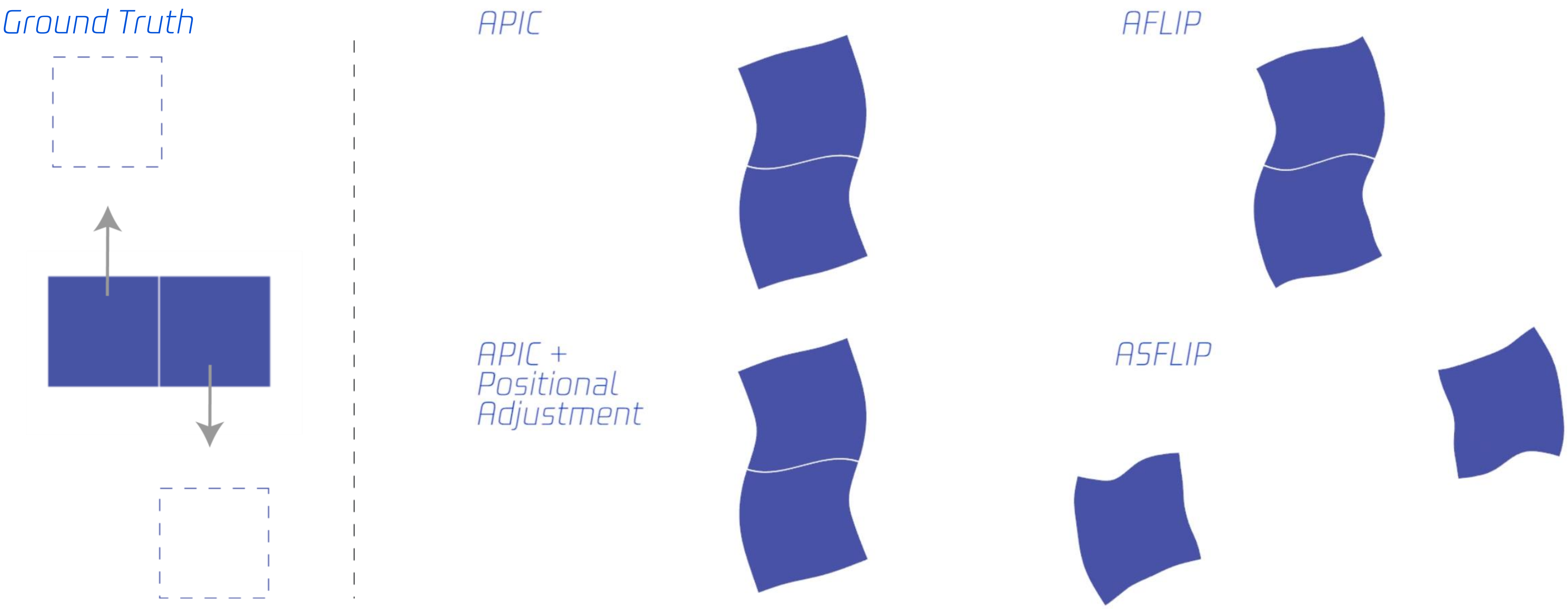
$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \Delta t \left[\sum_i w_{ip} \boldsymbol{v}_i^* + \beta_p \alpha \left(\boldsymbol{v}_p^n - \sum_i w_{ip} \boldsymbol{v}_i^n\right)\right]$$

Improves upon cases where continuum assumption no longer applies, preserving sub-grid movements

$$\beta_p = \begin{cases} 0, & \text{in boundary or source} \\ \beta_{\min}, & J_p < J_p^c \\ \beta_{\max}, & J_p \geq J_p^c \end{cases}$$


PIC
APIC
FLIP 0.99
FLIP 0.995
Full FLIP
NFLIP 0.99
NFLIP 0.995
ASFLIP 0.99
ASFLIP 0.995
Ground Truth
1.5Δx

2D FINITE ELEMENT SQUARES



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MORE EXAMPLES

WATER, SAND, HAIRS AND CLOTHES

paused

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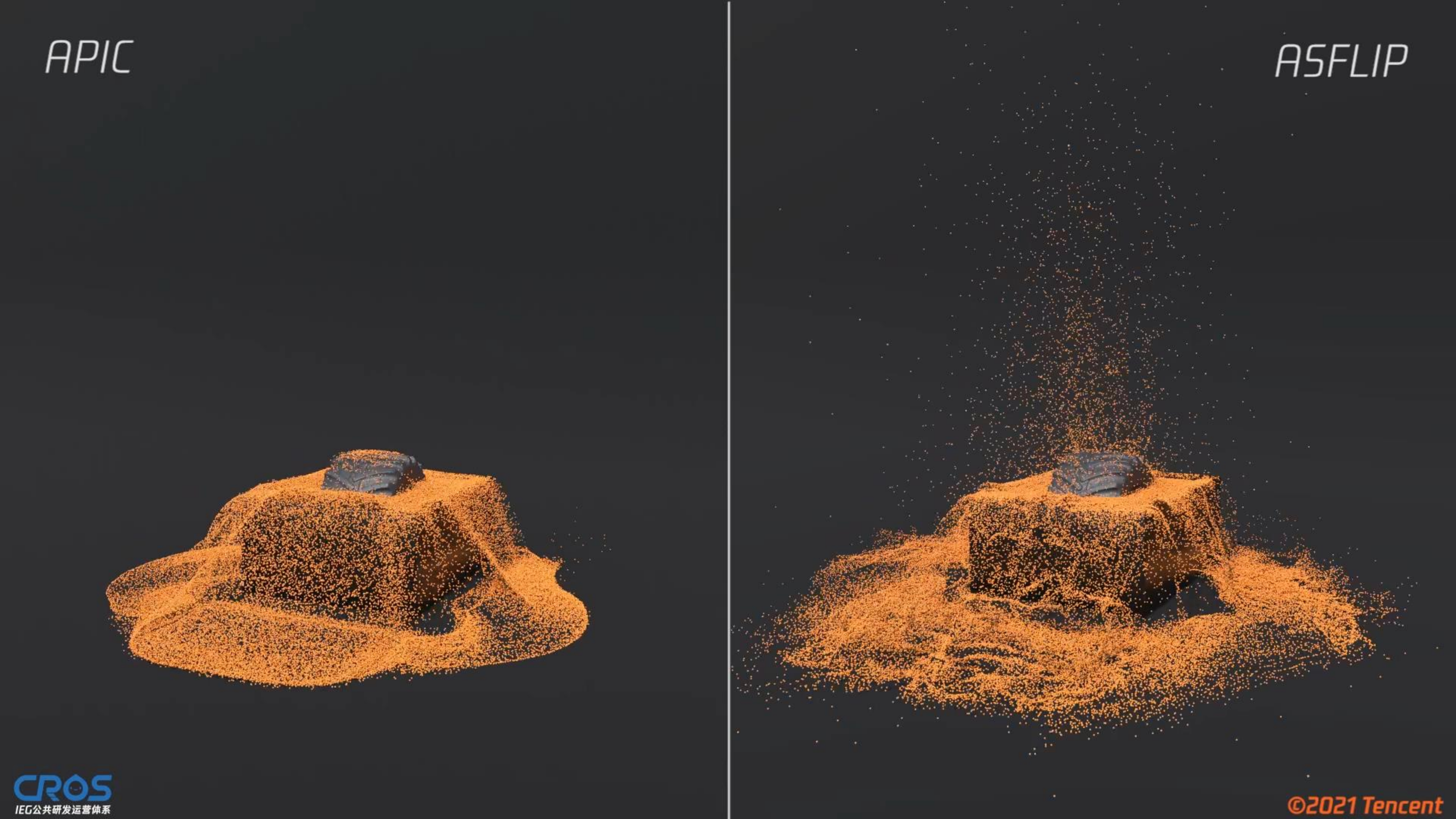
APIC
ASFLIP
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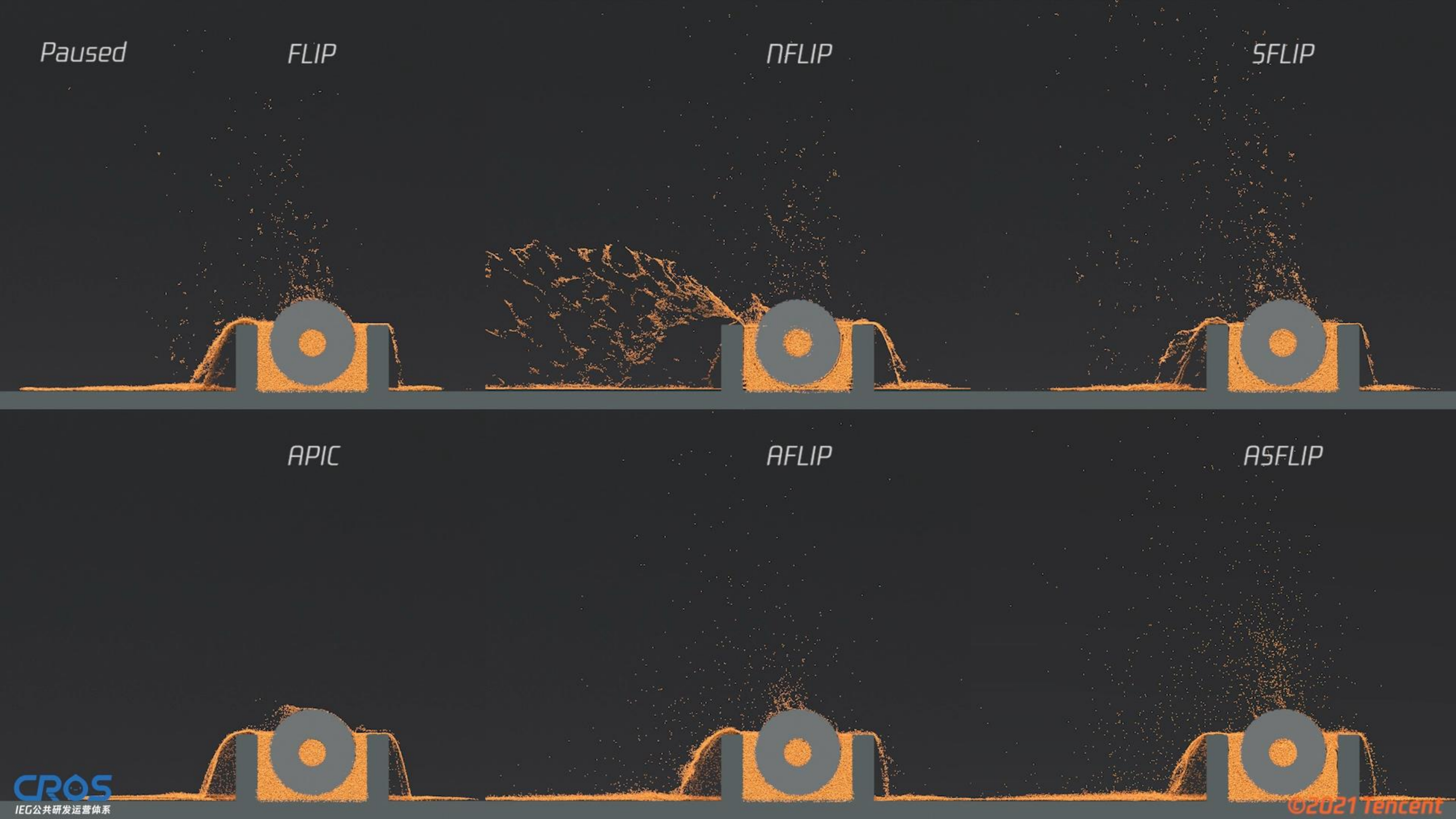

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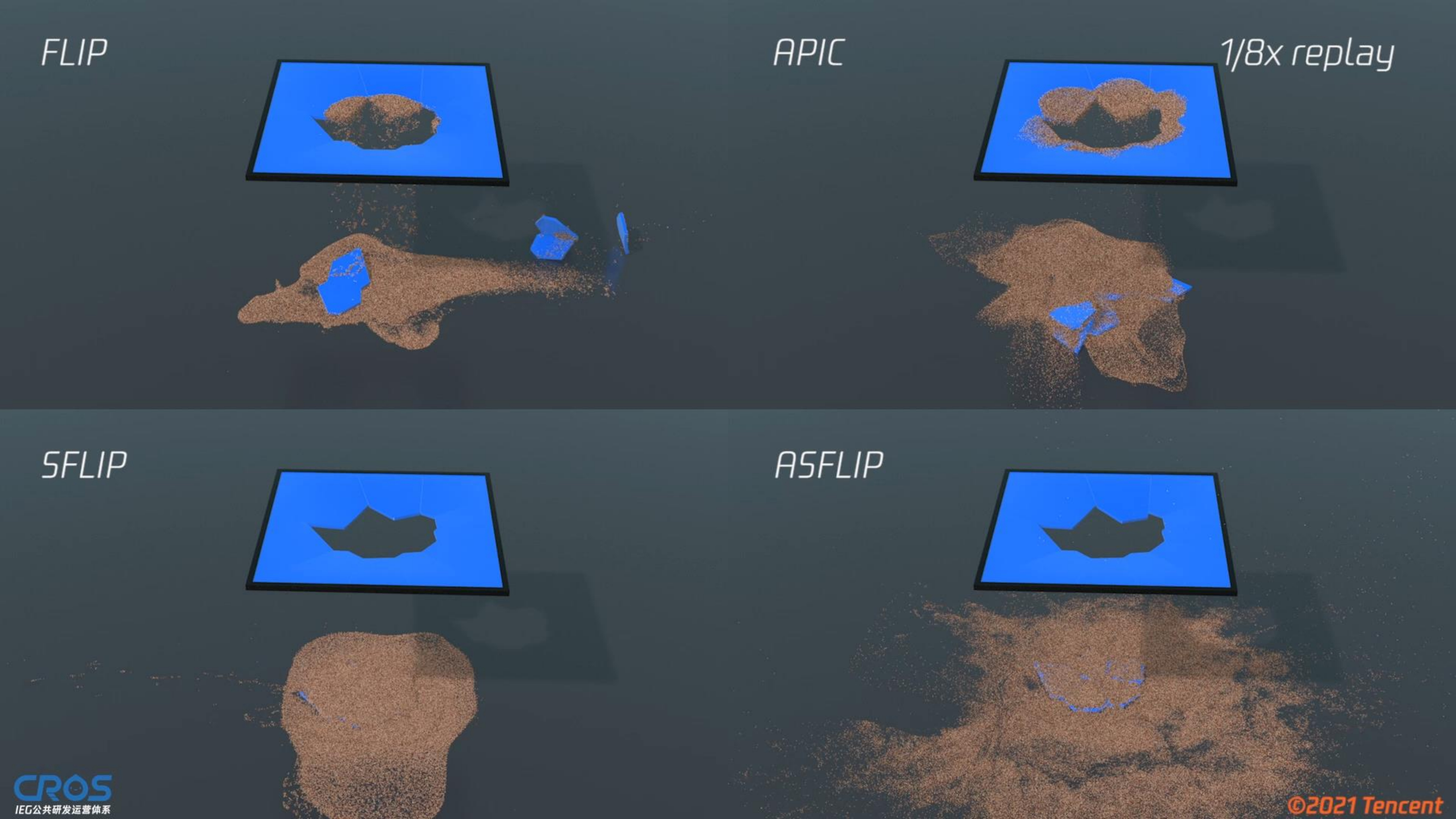
FLIP
APIC
1/8x replay
SFLIP
ASFLIP
CROS
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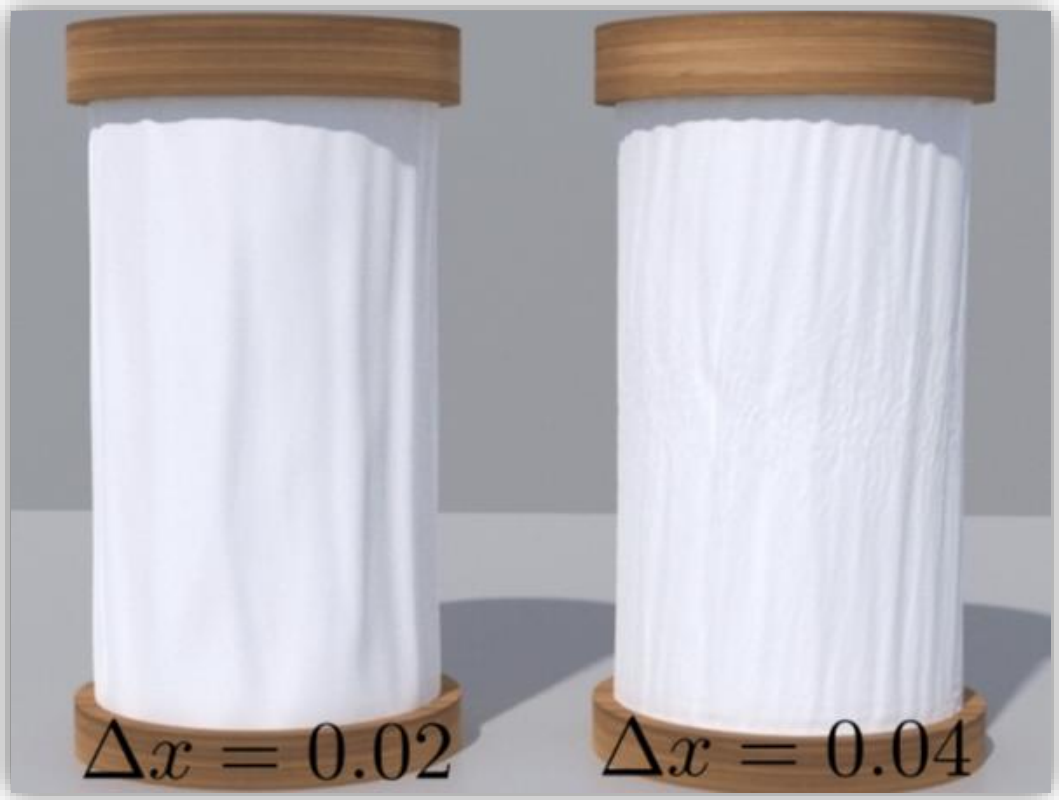
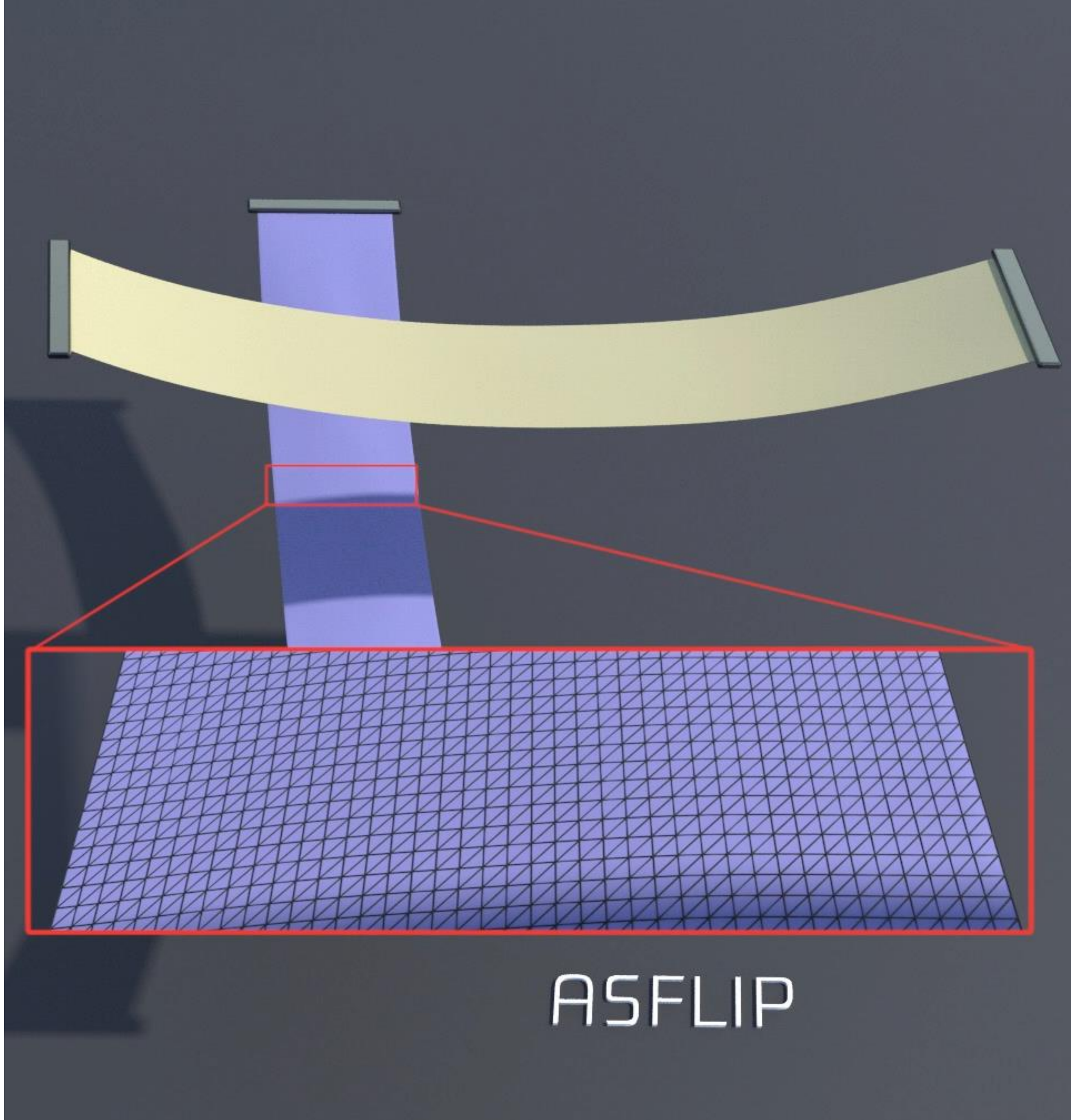
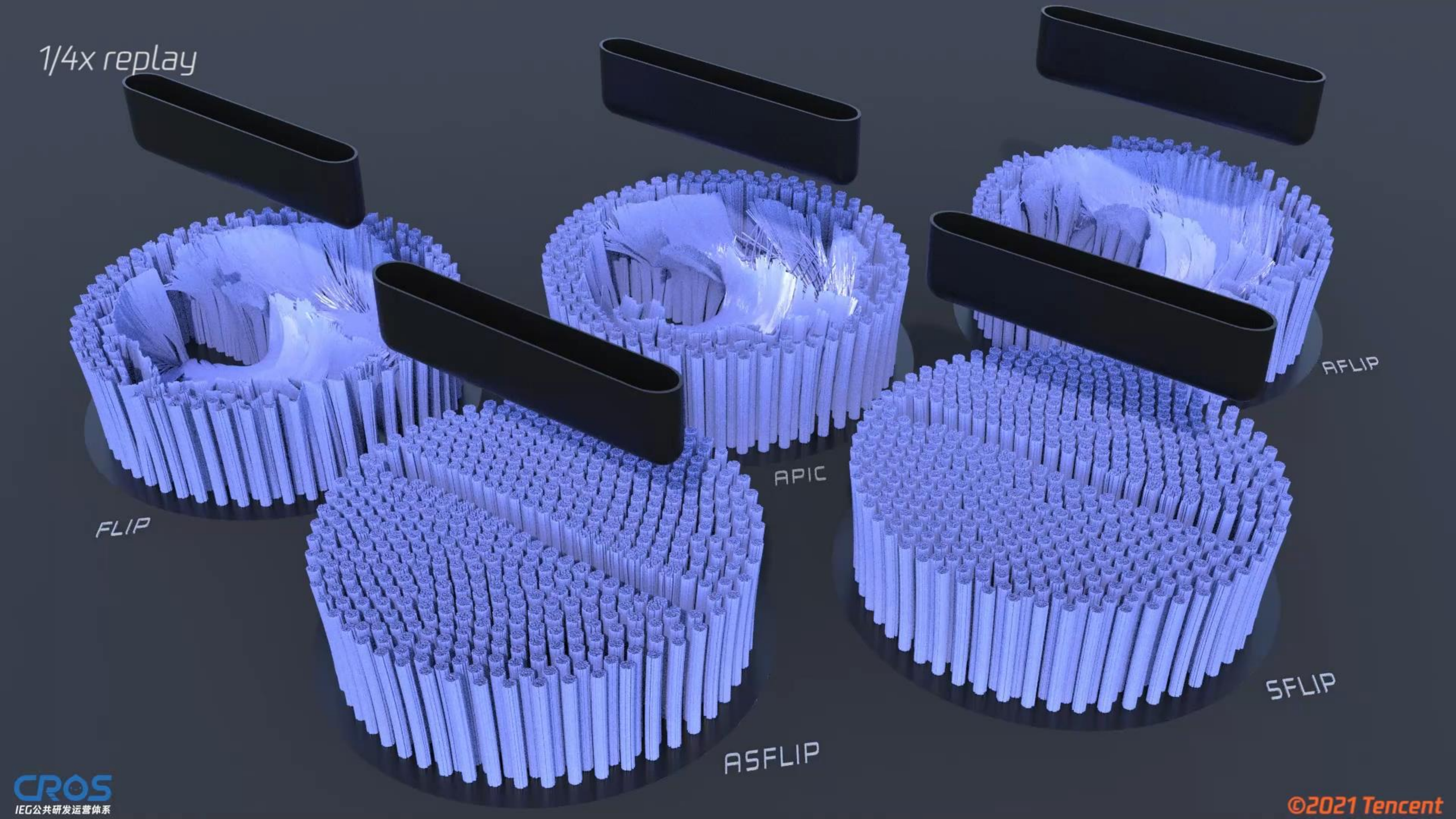


Grid-resolution dependent wrinkling
[Guo et al. 2018]




1/4x replay
FLIP
APIC
AFLIP
ASFLIP
SFLIP
CROS
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FLIP
APIC
SFLIP
NExT STUDIOS
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SUMMARY

SIGGRAPH 2021

Integrators	High-frequency motion	Undamped affine motion	Handle Boundary Condition	Easy Separation	Energetic Level
PIC	✗	✗	✓	✗	★
APIC	✗	✓	✓	✗	★★
FLIP	✓	✗	✓	✗	★★
NFLIP	✓	✗	✗	✗	★★
SFLIP	✓	✗	✓	✓	★★
AFLIP	✓	✓	✓	✗	★★★
ASFLIP	✓	✓	✓	✓	★★★

FUTURE WORK

- Numerical volume gain
- Simulators based on Chorin's projection
- Determine separation through fracture mechanics
- More accurate (self-) contact resolution
- Coupling with principled, discrete physics

PRE-PRINT, VIDEO, SOURCE CODE

http://yunfei.work/asflip/

TENCENT GAME AI RESEARCH CENTER